Large solutions in the context of fractional PDEs

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In the theory of elliptic PDEs, "large" solutions are those prescribed to present an explosive behaviour at the boundary of the underlying domain. These are a peculiarity of nonlinear equations and they are typically meant to provide with pointwise estimates, universal barriers, and Liouville-type results on all other solutions, but they pose nontrivial problems as to comparison principles, asymptotic behaviour, and uniqueness: this is due to the subtleties entailed by the behaviour of the nonlinearity at infinity and the lack of some usual tools such as a clear functional framework. In this talk we will review how the picture in the nonlocal/fractional framework is completely different, as large solutions become a fundamental part already of the linear theory.

Some important facts related with the notion of fractional convexity

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The main goal of this talk is to summarize some important aspects related with the concept of nonlocal fractional convexity that appear when we studied the nonlinear non-local operator

$$\Lambda_1^s u(x) := \inf_{\theta\in\mathbb{S}^{N-1}} \int_{\mathbb{R}} \frac{u(x+\tau\theta)-u(x)}{|\tau|^{1+2s}} d\tau,\ 0<s<1,$$

usually called *fractional first eigenvalue operator* or *first fractional truncated Laplacian*. We show the fast decayment of the viscosity solutions of the parabolic problem with Dirichlet conditions by studying the spectral theory of $\Lambda_1^s u$ and by proving a regularity up to the boundary result. Moreover we study the behavior of the fractional convexity when the fractional parameter goes to 1. The results have been obtained in collaboration with A. Quaas, L. Del Pezzo and J. Rossi. Part of the results that will be presented have been obtained in collaboration with D. Gomez-Castro and J.L. Vazquez (Madrid).

Some results on boundary value problems for Choquard equations

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We study the following nonlinear Choquard equation

$$-\Delta u + Vu = (I_\alpha * |u|^p)|u|^{p-2}u \ \text{ in } \Omega \subset \mathbb{R}^N,$$

where $N \geq 2$, $p \in (1, +\infty)$ and $V(x)$ is a continuous radial function such that $\inf_{x\in\Omega} V > 0$. First, assuming to have Neumann or Dirichlet boundary conditions, we prove existence of a positive radial solution when Ω is an annulus, or an exterior domain of the form $\mathbb{R}^N \setminus \overline{B}_a(0)$. We also provide a nonexistence result: if $p \geq \frac{N+\alpha}{N-\alpha}$ the corresponding Dirichlet problem does not have any nontrivial regular solution in strictly star-shaped domains. Finally, when considering annular domains, letting $\alpha \to 0^+$ we obtain an existence result for the corresponding local problem with power-type nonlinearity. This talk is based on a joint work with A. Cesaroni.

Soliton solutions for quasilinear modified Schrödinger equations

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We investigate the existence of soliton solutions of quasilinear modified Schrödinger equations. Since the classical Laplacian is replaced by an operator which admits coefficients depending on the solution itself, a classical variational approach does not work. Anyaway, by means of approximation arguments on bounded sets and following some ideas which exploit the interaction between two norms, the related functional admits a critical point in a "good" Banach space which is a soliton solution of the given problem. These results are part of joint works with Giuliana Palmieri, Addolorata Salvatore and Caterina Sportelli.

Sobolev regularity of the stress-field in nonlinear elliptic problems

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Second-order regularity results are established for solutions to second-order elliptic equations and systems, in divergence form, with principal part having Uhlenbeck structure and square-integrable right-hand sides. Such a regularity amounts to the membership in a Sobolev space of the so-called stress-field associated with the elliptic operator. In the case of equations, differential operators depending on anisotropic norms of the gradient are also included. Both local and global estimates are obtained. Global estimates concern solutions to homogeneous Dirichlet problems under minimal regularity assumptions on the boundary of the domain. In particular, no regularity of its boundary is needed if the domain is convex. A critical step in the approach is a sharp pointwise inequality for the involved elliptic operator. This talk is based on diverse joint investigations with C.A.Antonini, A.Kh.Balci, G.Ciraolo, L.Diening, A.Farina, and V.Maz'ya.

Propagation phenomenon in some integrodifferential equations with a Levy diffusion

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I will present some recent results on the description of propagation phenomenon in a homogeneous environment that are encountered in some nonlocal reaction diffusion models whose diffusion process is described by a Lèvy diffusion. In the literature, such models have been studied for mainly two classes of Lèvy processes: the alpha stable leading to consider fractional Laplacian operators and the Compound Poisson Process, leading to consider convolution type operator. Recently, travelling fronts vs. acceleration phenomena have been extensively studied for these two types of processes and various shapes of nonlinearities. I will present recent generalisations of such dichotomy existence of front solution vs acceleration phenomenon for various types of nonlinearity in a context of a more generic Lèvy process, i.e. process leading to consider absolutely continuous jump measures $d\mu(z) = J(z)\,dz$ such that J satisfies $\int_R \min\{1, z^2\}\, J(z)dz < +\infty$.

Constant sign and sign changing NLS ground states on metric graphs

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In this talk, we investigate existence and nonexistence of positive and nodal action ground states for the nonlinear Schr¨odinger equation on metric graphs. For noncompact graphs with finitely many edges, we detect purely topological sharp conditions preventing the existence of ground states or of nodal ground states. We also investigate analogous conditions of metrical nature. The negative results are complemented by several sufficient conditions to ensure existence, either of topological or metrical nature, or a combination of the two. This is based on joint work with Simone Dovetta (Politecnico di Torino (Italy)), Damien Galant (UPHF et UMons (Belgium)), Enrico Serra (Politecnico di Torino (Italy)), Christophe Troestler (UMons (Belgium)).

Existence and non-existence results for non-linear elliptic systems involving hardy potential

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The main goal of this work is to study the existence and the non-existence of a non-negative super-solution to class of gradients-potential systems with Hardy term. More precisely, we consider the system

$$\begin{cases} -\Delta u - \lambda \frac{u}{|x|^2} = f_1(x, v, \nabla v) & \text{in } \Omega, \\ -\Delta v - \lambda \frac{v}{|x|^2} = f_1(x, u, \nabla u) & \text{in } \Omega, \\ u = v = 0 \quad \text{on } \Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$, $N \geq 3$, is a bounded regular domain such that $0 \in \Omega$. Here, $0 < \lambda \leq \left(\frac{N-2}{2}\right)^2$. We consider two cases:

- First Case: $f_1(x, v, \nabla v) = v^p$ and $f_2(x, u, \nabla u) = |\nabla u|^q$.
- Second Case: $f_1(x, v, \nabla v) = |\nabla v|^p$ and $f_2(x, u, \nabla u) = |\nabla u|^q$.

For $p, q > 1$, we prove the existence of an optimal critical curve in the (p, q)-plane, that seprates the existence and non-existence regions.

A one-sided two phase Bernoulli free boundary problem

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We study a two-phase free boundary problem in which the two phases satisfy an impenetrability condition. Precisely, we have two ordered positive functions, which are harmonic in their supports, satisfy a Bernoulli condition on the one-phase part of the free boundary and a two-phase condition on the collapsed part of the free boundary. For this two-membrane type problem, we prove an epsilon-regularity theorem in all dimensions. Then, in dimension two, we study the fine structure of the free boundary, showing that the branching points can only occur in finite number. This is a joint work with Luca Spolaor and Bozhidar Velichkov.

Continuity of solutions to equations with weakly singular nonlocal operators

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Partial differential equations involving the Laplacian exhibit a regularization effect, leading to smoother solutions. The regularizing effect is directly related to the order of the operator and this idea translates well to more general operators. In particular, this effect extends to the fractional Laplacian, a nonlocal operator prototype. In the theory of nonlocal operators the order of the operator is related to an associated singular kernel. In this talk, I discuss the question on how weakly singular the kernel can be to ensure the continuity of solutions. The results are part of a joint project with Moritz Kassmann and Tobias Weth.

Asymptotic expansions and finite difference schemes for the fractional p-Laplacian

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We propose a new asymptotic expansion for the fractional p-Laplacian with precise computations of the errors, based on the idea that the singular part of the integral representation of the operator behaves like a local p-Laplacian with a weight correction. We also propose monotone finite difference approximations with explicit weights, and we obtain the associated error estimates.

Nonexistence and existence of solutions for nonautonomous critical equations of Schrödinger type

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This talk deals with nonautonomous elliptic equations on R^N, involving the p-laplacian and a nonlinearity that is critical from the viewpoint of the Sobolev embedding. Some recent results about the nonexistence, existence, and multiplicity of solutions are presented, depending on the shape of the potential. A new definition of barycenter and concentration rate of functions is presented. Joint work with Carlo Mercuri.

Normalized solutions and limit profiles of the Gross-Pitaevskii-Poisson equation

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Gross-Pitaevskii-Poisson (GPP) equation is a nonlocal modification of the Gross-Pitaevskii equation with an attractive Coulomb-like term. It appears in the models of self-gravitating Bose-Einstein condensates proposed in cosmology and astrophysics to describe Cold Dark Matter and Boson Stars. We investigate the existence of prescribed mass (normalised) solutions to the GPP equation, paying special attention to the shape and asymptotic behaviour of the associated mass-energy relation curves and to the limit profiles of solutions at the endpoints of these curves. In particular, we show that after appropriate rescalings, the constructed normalized solutions converge either to a ground state of the Choquard equation, or to a compactly supported radial ground state of the integral Thomas-Fermi equation. In different regimes the constructed solutions include global minima, local but not global minima and unstable mountain-pass type solutions. This is a joint work with Riccardo Molle and Giuseppe Riey.

The Talenti comparison result in a quantitative form

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We obtain a quantitative version of the classical comparison result of Talenti for elliptic problems with Dirichlet boundary conditions. The key role is played by quantitative versions of the Pólya-Szego inequality and of the Hardy-Littlewood inequality. This is a joint work with V. Amato, R. Barbato and A.L. Masiello (Federico II).

Blow-up and global solutions for a parabolic problem with Trudinger-Moser nonlinearity

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We consider the Cauchy problem for a 2-space dimensional heat equation with exponential nonlinearity. More precisely, we consider initial data in $H^1(\mathbb{R}^2)$, and a square-exponential nonlinearity, which is critical in the energy space $H^1(\mathbb{R}^2)$ in view of the Trudinger-Moser inequality. By means of energy methods, we discuss the dichotomy between blow-up and global existence for solutions below the ground state energy level. The splitting between blow-up and global existence for low energies is determined by the sign of a suitable functional, and it is related to the corresponding Trudinger-Moser inequality. This is a joint work with Michinori Ishiwata (Osaka University), Bernhard Ruf (Istituto Lombardo Accademia di Scienze e Lettere), and Elide Terraneo (Università degli Studi di Milano).